Damages Analysis and the Cost of Equity Capital Size Premium

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Damages analysts routinely develop damages measurements that include income projections and apply a present value discount rate. Such damages measurements often include lost profits method analyses. Such damages measurements also include cost to cure method analyses and reasonable royalty rate method analyses. One of the typical components of the discount rate measurement in the damages analysis is the estimation of cost of equity capital. The measurements of many of the cost of equity capital components are typically not controversial in the damages measurement discount rate calculation. The measurement of the size risk premium component of the cost of equity capital can sometimes result in a disagreement among practitioners with regard to the discount rate calculation. This discussion summarizes many of the damages analyst considerations with regard to the measurement of the equity size risk premium. This equity size risk premium is one component of the present value discount rate calculation that is developed as part of the damages measurement analysis.

Introduction

Damages analysts routinely develop damages measurements that include income projections and apply a present value discount rate. Such damages measurements include lost profits method analyses. Such damages measurements also include cost to cure method analyses and reasonable royalty rate method analyses.

One typical component of the discount rate measurement in the damages analysis is the cost of equity capital estimation. The measurements of many of the cost of equity capital components are typically not controversial in the damages measurement discount rate calculation.

The measurement of the size risk premium cost of equity capital component can sometimes result in disagreement with regard to the discount rate calculation. This discussion summarizes the damages analyst considerations with regard to the measurement of the equity size risk premium. This equity size risk premium is one component of the present value discount rate calculation developed as part of the damages measurement analysis.

The cost of equity capital is a foundational component of the present value discount rate used in many lost profits method analyses and many other damages measurement analyses. Some of the generally accepted cost of equity capital estimation models applied in the damages measurement process include the build-up rate model and the modified capital asset pricing model ("MCAPM").[1]

As a component of these generally accepted models, analysts often include a size risk premium—or alpha adjustment factor—as part of the cost of equity estimation procedure.

This discussion considers the following topics:

1. Empirical evidence supporting the size premium adjustment
2. Observations regarding the size premium

3. Observations regarding the Center for Research in Security Prices ("CRSP") size premium 10th decile category
4. Liquidity issues that may account for the size premium

A typical formula for the build-up model ("BUM") to estimate the cost of equity capital follows:

$$K_e = R_f + ERP + IRP + SRP + \alpha$$

where:

K_e = Cost of equity capital
R_f = Risk-free rate of return
ERP = Long-term equity risk premium
IRP = Industry-related equity risk premium
SRP = Size-related equity risk premium
α = Unsystematic equity risk premium

There is a general consensus among damages analysts as to the appropriate risk-free rate of return to use in the BUM. Damages analysts typically select and apply the market yield on the 20-year U.S. Treasury bond as the risk-free rate of return component.

For certain damages analyses, the investment duration may be less than 20 years and an analyst may select a risk-free rate of return with an investment duration commensurate with the specific investment duration.

The selected long-term equity risk premium ("ERP") is not as consistently applied among analysts. Certain damages analysts advocate the use of a more normalized equity risk premium, of say 5 percent. Other analysts elect to use the variables included in the CRSP Decile Size Premium Study published in the *2017 Valuation Handbook – U.S. Guide to Cost of Capital* ("*Valuation Handbook*") in Appendix 3.[2]

The *Valuation Handbook* ERP data are typically cited, providing an estimated ERP premium of around 6 percent.

Other components of the BUM cost of equity estimate often include an industry-related equity risk premium, a size-related equity risk premium, and an unsystematic equity risk premium. By adding an industry-related risk premium, general industry risk is incorporated in the cost of equity.

This general industry risk premium is not specifically addressed in the long-term equity risk premium component. The industry risk component of the build-up cost of equity capital incorporates systematic risk, in much the same way that beta incorporates industry risk in the capital asset pricing model ("CAPM").

The next two components of the BUM are the size-related equity risk premium and the unsystematic equity risk premium. An overview of the size-related equity risk premium is presented later in this discussion.

The unsystematic equity risk premium component is often applied by analysts. This component is used to incorporate risk that is specific to the subject investment—that is, lack of management talent, potential labor issues specific to the damaged party, potential of losing a key client or key personnel, and/or potential cost/risk not identified in financial projections, and so forth.[3]

The basic CAPM formula for estimating the cost of equity capital for publicly traded security analysis follows:[4]

$$K_e = R_f + [\beta \times ERP]$$

where:

K_e = Cost of equity capital
R_f = Risk-free rate of return
β = Industry beta
ERP = Long-term equity risk premium

Damages analysts apply many of the same components in the CAPM formula that are used in the BUM. That is, it is typical for damages analysts to rely on the same risk-free rate of return and long-term equity risk premium component factors when applying both the BUM and the CAPM to estimate the cost of equity. The one distinguishing CAPM factor is beta.[5]

Beta, in general terms, is used to incorporate market risk (general equity risk and industry risk) in an equity cost of capital estimate. As a best practice, it is often important to examine multiple lookback periods and frequencies when determining a beta estimate.

Ultimately, the goal of the damages analyst should be to estimate a beta that fairly represents the systematic risk and stock price variability of the subject company as compared to the broad equity market, over a relevant time period. The damages analyst should keep in mind that the beta estimate is the mean of a statistical distribution that results from a regression analysis.

Some of the factors that a damages analyst may consider when examining multiple beta estimates include the following:

1. The mean of each distribution
2. The relationship between the means of each distribution
3. The dispersion about the mean for each distribution
4. The relationship between the dispersions about the means of each distribution

Further adjustments to CAPM may include:

1. the size-related equity risk premium component and
2. the unsystematic equity risk premium component.

By including these alpha adjustments, the CAPM becomes the MCAPM.

The MCAPM formula for estimating the cost of equity capital for use in an income-related damages measurement analysis is presented as follows:

$$K_e = R_f + [\beta \times ERP] + SRP + \alpha$$

where:

K_e	=	Cost of equity capital
R_f	=	Risk-free rate of return
β	=	Industry beta
ERP	=	Long-term equity risk premium
SRP	=	Size-related equity risk premium
α	=	Unsystematic equity risk premium

The MCAPM and the BUM provide generally consistent and easy to replicate cost of equity capital calculations.

Size Risk Premium and Why It Should Be Applied

Based on empirical observation, it is generally accepted that small companies present a greater investment risk than larger companies do. Therefore, smaller companies typically have a greater cost of capital than do larger companies. In other words, there is a significant (negative) relationship between size and historical equity returns.

It is also generally accepted that small companies have certain risk characteristics that are more prevalent than in larger companies.

These small company risk characteristics include the following:

1. Potential competition issues (it is easier to enter the market and compete with small companies, while larger companies have resources to mitigate competitive challenges)
2. Economic issues and concern (larger companies can better cope with economic downturns than small companies)
3. Limited access to capital (small companies can find it difficult to obtain funding while larger companies typically have more options for funding)
4. Management depth concerns (large companies do not have key employee concerns in the same way that smaller companies do)
5. Customer concentration and product concentration risk (small companies are typically not as diversified in product offerings and are often beholden to a small group of customers)
6. Liquidity concerns and lack of market coverage (small companies do not enjoy the same level of analyst coverage and small company stock is typically less liquid than larger companies)

Rolf Banz, in a 1981 study, is credited and commonly cited for his research focusing on the empirical relationship between equity return and the total market value of NYSE common stocks.

According to Banz, smaller firms have higher risk-adjusted returns, on average, than larger firms. For the approximately 40 years covered in the study, on average, small firms recorded larger risk-adjusted returns than large firms traded on the NYSE. The Banz study found that the size effect did not exhibit linear attributes; however, the size effect was found to be more pronounced in smaller firms.

Another noteworthy finding in the Banz study was that the study suggests no theoretical foundation for the size effect. It concluded no determination as to whether the size effect factor is due to size itself or whether size is just a proxy for one or more true but unknown factors correlated with size. According to Banz, the size effect exists but it is not clear why it exists.

The Kroll—formerly Duff & Phelps—Cost of Capital Navigator is one reference source for measuring the size risk premium adjustment. The Cost of Capital Navigator provides empirical evidence of the size premium phenomena. The Cost of Capital Navigator is a web-based resource that provides certain cost of equity capital components.

The Cost of Capital Navigator defines the size premium as the difference between actual historical excess returns and the excess return predicted by beta (referred to as the "CRSP size premium").[8]

Exhibit 1 presents empirical evidence of the CRSP size premium, as published in the most recent *Valuation Handbook*.[9] As presented in Exhibit 1, the empirical data illustrates stock market returns by size decile for the 1926 to 2020 time period.[10]

Exhibit 1
Current 10 Decile Statistics
As of December 31, 2020

Decile	Market Capitalization (in $ millions)	Geometric Mean (%)	Arithmetic Mean (%)	Standard Deviation (%)
1 - Largest	29,025.8 to 1,966,078.9	9.67	11.39	18.77
2	13,178.7 to 28,808.1	10.73	12.93	21.22
3	6,743.4 to 13,177.8	11.18	13.65	23.06
4	3,861.9 to 6,710.7	10.99	13.85	25.19
5	2,445.7 to 3,836.5	11.44	14.48	25.79
6	1,591.9 to 2,444.7	11.49	14.84	26.72
7	911.6 to 1,591.8	11.82	15.53	28.62
8	452.0 to 911.1	11.43	15.84	32.37
9	190.0 to 451.8	11.67	16.91	36.50
10 - Smallest	2.2 to 189.8	13.3	20.04	41.69

Source: Kroll Cost of Capital Navigator

The annual stock market returns are separated into 10 deciles based on market capitalization. As the deciles get smaller, from 1 to 10, the historical stock market returns increase. The standard deviation of stock return portfolios also increases as the deciles get smaller.[11] This increase in the standard deviation reflects noise in the data.

A review of Exhibit 1 indicates that the most statistical data noise in the 10 decile stratification is in the 10th decile classification.

Other empirical evidence, in support of the small capitalization size premium adjustment, is provided by international equity market data. For example, in the United Kingdom, a study conducted using its equity markets concluded a small capitalization stock premium of around 7 percent.[12]

The U.K. study was conducted using equity market data from 1955 to 1984.

In 2015, an equity risk premium analysis study of small capitalization stocks in 23 global markets was conducted by Dimson, March, and Staunton.[13]

In the 23 global equity markets, small cap stocks outperformed in every market except for Norway, Finland, and the Netherlands. In general, evidence of the small capitalization stock premium is more prevalent in developed markets than in emerging markets.

Size Risk Premium and Certain Factors to Consider

There are several observations regarding the data used to calculate the size risk premium adjustment. A few of these observations include the following:

- The small capitalization premium has disappeared in recent years.
- A premium is unduly influenced by stocks with less than $5 million in market capitalization.
- The supporting data are too noisy to calculate a meaningful size premium estimate due to the evidence of significant standard errors and seasonality.
- There may be other factors than size that contribute to greater small capitalization stock returns compared to large capitalization stock returns, such as:
  - bid/ask spread bias,
  - delisting bias,
  - transaction costs, and
  - liquidity.

It is generally accepted that the small capitalization stock premium was observable prior to 1980. However, it appears that the small capitalization stock premium has decreased since 1981.[14]

The Horwitz study found that during the period of 1963 to 1981, the annualized return difference between small and large firms was greater than 13 percent.[15]

However, the study also found that, during the period of 1981 to 1997, the annualized difference was negative 2 percent.[16]

Perhaps the reason for the small capitalization risk premium decrease is twofold:

1. Market corrections induced by investor understanding of the small capitalization premium phenomena
2. External economic and technological changes in the way the securities are bought and sold

As suggested in the Horowitz study, a trend toward passive investing using index funds that give more weight to large capitalization stocks may be a reason for increases in capital gain performance of large capitalization stocks.[17]

Because small capitalization stock performance as compared to large capitalization stock performance over short-term duration is typically more erratic, measurement over a longer term is preferred. For holding measurement periods of 1 year, 5 years, 10 years, 20 years, and 30 years, small capitalization stocks outperform large capitalization stocks a majority of the time—measured from 1926 to 2016.[18]

As the measurement period increases, so does the likelihood of small capitalization stock outperformance of large capitalization stocks.

Small capitalization stock performance is cyclical, and cyclicality should be expected. Small capitalization stock returns are variable and somewhat volatile. According to one analyst, if small companies always earned more than large companies, then small companies would not be a riskier investment endeavor in the aggregate.[19]

It is also noteworthy that bond prices occasionally outperform equities. In 2014, long-term U.S. government bonds outperformed the S&P 500 Index by 10 percent.[20]

Even over a long period of time, which provides the strongest support for the existence of a small cap premium, the Horowitz study found that removing stocks with less than $5 million in market capitalization causes the small firm effect to vanish.[21]

According to the Horowitz study, the percentage of companies with stock prices of less than $2 per share was greater in the period of 1982 to 1997 than in the period of 1963 to 1981.[22]

In the smallest decile, 11.7 percent of companies traded at prices less than $2 a share between 1963 to 1981. In the 1982 to 1997, the percentage of companies traded at prices less than $2 per share in the smallest decile was 29.7 percent.

In general, historical equity returns exhibit unpredictable variability. Estimates of security risk using historical equity returns reflect noise in the form of large standard errors.[23]

As presented in Exhibit 1, as decile classifications of stock increase—correlated with smaller capitalization stocks—the standard deviation increases. The standard errors by decile class suggest that the small capitalization premium is fragile—almost to the point of lacking statistical significance.[24]

The January effect, seasonality of small capitalization stock returns, is a well-documented phenomenon. The January effect is described as the empirical observation that rates of return for small stocks have, on average, performed better in January than in other months of the year.[25]

In the Horowitz study, the average monthly return in the month of January for small capitalization stocks was 10.20 percent as compared to 0.73 for the average monthly return for February to December.[26]

The Horowitz study calculated the premium using NYSE, AMEX (now NYSE MKT), and Nasdaq stock returns for the period of 1963 to 1997. Other studies have reached similar conclusions. Although the January effect is interesting, it does not disprove that a size premium exists.

It is an unsettled discussion point that the bid/ask spread adds a certain bias to stock returns.[27] This observation is primarily focused on less liquid companies that have larger bid/ask spreads.

Most of the small-size effect studies (such as the SBBI equity study previously prepared by Morningstar, the CRSP equity study previously prepared by Duff & Phelps, and the CRSP equity study now prepared by Kroll) use the CRSP database, which relies on the closing stock price to measure rates of return.

For thinly traded stocks, the ask price is not always a realistic price. Because the small-size effect studies measure size using portfolio returns calculated on a monthly basis, one publication suggests the bid/spread bias issue has only a trivial impact on the small stock risk premium.

Some observers suggest that a delisting bias exists in the Morningstar decile size premium calculations due to its use of the CRSP database without adjustment.[28]

The reason for this possible bias is because the CRSP database information is allegedly missing prices for certain securities in the period immediately after these certain securities are delisted from a stock exchange.

According to the CRSP, as concluded in a CRSP white paper, the so-called delisting bias is greatly exaggerated.[29]

A few observers have suggested that the size effect is not relevant because various studies have ignored transaction costs in measuring rates of return.[30]

The primary observation is that small capitalization stocks often have higher transaction costs than large stocks. Because of the higher transaction costs for small capitalization stocks, it is possible that the historical small-stock-related size risk premium would be reduced if transaction costs and holding periods were factored into the measurement of rates of return.

As published in the *Cost of Capital*, 5th edition, Ashok Abbott prepared a study of transaction costs by decile for securities listed on the NYSE, AMEX, and the Nasdaq from January 1993 to December 2008. The securities trading cost was estimated as the difference between the daily holding return (closing price to closing price) and the daily trading return (ask price from the previous day to the bid price of the current day).

As presented in Exhibit 2, as company size decreases, the average daily trading cost, as a percentage of the trade, increases. The study found that larger firms are traded at lower costs and are subject to less pricing pressure than smaller firms.

Abbott also prepared an analysis of trading costs as differentiated by liquidity. The results of the Abbott study suggest that as company liquidity decreases, trading costs increase. Another notable finding of the Abbott study indicates that the least liquid stocks comprise the smallest market capitalization size-related decile.

Exhibit 3 presents the Abbott study analysis of liquidity and trading costs.

A discussion of stock liquidity and the equity size premium is presented in more detail below.

CRSP Size Premium 10th Decile Category Considerations

The companies that comprise the CRSP size premium 10th decile category have equity market capitalizations that range from $2.2 million to $189.8 million. As of December 31, 2020, the risk premium related to the companies comprising the 10th decile was 5.01 percent.[31]

The companies that comprise the CRSP size premium 10th decile are broken down into subcategories 10a and 10b, as presented in the Cost of Capital Navigator. The companies that comprise the 10a subdecile include companies with market capitalizations between $96.6 million and $189.8 million, and the reported size premium is 3.49 percent.[32]

The companies that comprise the 10b subdecile include companies with market capitalizations between $2.2 million and $95.2 million, and the reported size premium is 8.12 percent.[33]

Within the 10a subdecile and 10b subdecile categories of the 10th decile, the Cost of Capital Navigator presents more subcategories. The 10a subdecile is broken into 10w and 10x subdeciles, while the subdecile 10b is broken into 10y and 10z.

Exhibit 4 presents the Cost of Capital Navigator, CRSP size premium 10th decile subdecile category market capitalizations, and size risk premiums subcategory breakdown.

As presented in Exhibit 4, companies that are classified in the CRSP size premium 10th decile vary considerably in market capitalization and applicable size risk premium. The size risk premium ranges from 2.60 percent to 11.29 percent, a spread of 8.69 percent, or 869 basis points.

As presented in Exhibit 4, as the size of the company increases, its size risk premium decreases. That is why it is important to correctly interpret and apply the size risk premium component of the MCAPM—assuming an analyst applies an equity size risk premium adjustment.

According to the Cost of Capital Navigator, "The CRSP Deciles Size Premia include all companies with no exclusion of speculative (e.g., start-up) or distressed companies whose market capitalization may be small because they are speculative or distressed."[34]

The distressed company issue can be seen through analysis of the 10th decile subcategories of 10y and 10z. For example, the average company in the 10y subcategory typically records a negative net income. In some years, the average of the decile subcategory 10y and 10z companies also recorded negative earnings, before, interest, taxes, depreciation, and amortization ("EBITDA").

Exhibit 5 presents financial statistics related to the CRSP size risk premium 10th decile subcategories 10y and 10z as published in the *Valuation Handbook* for 2014 and 2017, and in the Cost of Capital Navigator for 2020.

As presented in Exhibit 5, the companies that populate subcategory 10y and 10z are, on average, recording negative net income. In many cases, the companies that populate subcategory 10y and 10z are recording negative EBITDA.

Exhibit 2
Average Trading Costs by Market Value of Equity Decile
For the Period of January 1993 to December 2008

Market Value of Equity Portfolio	Average Daily Trading Cost
1 - Largest Companies	0.75489%
2	1.07736%
3	1.33369%
4	1.67466%
5	2.05954%
6	2.50398%
7	3.16594%
8	4.13995%
9	5.57523%
10 - Smallest Companies	9.67356%

Source: *Cost of Capital* , 5th ed., 367.

Exhibit 3
Average Trading Costs Based on Equity Liquidity
For the Period of January 1993 to December 2008

Decile by Liquidity	Average Daily Trading Cost
1 - Most Liquid Companies	1.48241%
2	1.82615%
3	2.02649%
4	2.15579%
5	2.28703%
6	2.47802%
7	2.73914%
8	3.03041%
9	3.73256%
10 - Least Liquid Companies	5.60277%

Source: *Cost of Capital* , 5th ed., 368.

Exhibit 4
10th Decile Subcategories
As of December 31, 2020

10th Decile Subcategory	Market Capitalization	Equity Size Premium
Decile 10w	$138.8 Million to $189.8 Million	2.60%
Decile 10x	$96.6 Million to $137.9 Million	4.65%
Decile 10y	$46.9 Million to $95.2 Million	6.60%
Decile 10z	$2.2 Million to $46.9 Million	11.29%

Source: Kroll Cost of Capital Navigator.

Exhibit 5
10th Decile Subcategories 10y and 10z
Statistics as of September 30, 2013, 2016, and 2019

	Percent of Subcategory	Market Value of Equity (in $ Millions)	Market Value of Invested Capital (in $ Millions)	Sales (in $ Millions)	5-Year Average Net Income (in $ Millions)	5-Year Average EBITDA (in $ Millions)
As of September 30, 2013:	95th Percentile	181.19	566.53	734.63	12.99	80.76
10th Decile Subcategory 10y	75th Percentile	161.62	227.93	233.67	5.47	22.95
Market Value of Equity Range	**50th Percentile**	**138.58**	**175.02**	**74.86**	**(1.71)**	**7.74**
$100.9 Million and $184.9 Million	25th Percentile	116.69	139.05	29.38	(15.95)	(7.13)
	5th Percentile	103.44	110.39	1.42	(71.07)	(30.51)
As of September 30, 2013:	95th Percentile	94.04	210.99	318.61	7.56	27.73
10th Decile Subcategory 10z	75th Percentile	70.49	95.17	78.89	1.81	6.62
Market Value of Equity Range	**50th Percentile**	**44.97**	**64.98**	**31.77**	**(1.42)**	**1.18**
$2.4 Million and $100.8 Million	25th Percentile	25.12	34.97	15.29	(8.25)	(4.43)
	5th Percentile	7.89	11.23	1.03	(33.57)	(17.97)
As of September 30, 2016:	95th Percentile	123.59	694.33	516.09	11.54	69.39
10th Decile Subcategory 10y	75th Percentile	109.94	198.68	151.97	4.86	17.89
Market Value of Equity Range	**50th Percentile**	**96.02**	**121.77**	**51.50**	**(1.50)**	**3.99**
$73.6 Million and $127.3 Million	25th Percentile	82.85	99.80	29.23	(16.28)	(10.61)
	5th Percentile	74.68	77.79	8.28	(37.15)	(22.00)
As of September 30, 2016:	95th Percentile	70.11	176.78	248.60	4.60	22.77
10th Decile Subcategory 10z	75th Percentile	53.10	72.14	67.03	0.71	3.18
Market Value of Equity Range	**50th Percentile**	**34.34**	**46.75**	**25.30**	**(3.96)**	**(1.55)**
$2.5 Million and $73.5 Million	25th Percentile	18.85	25.49	8.09	(13.93)	(9.47)
	5th Percentile	6.66	9.76	1.03	(25.15)	(18.67)
As of September 30, 2019:	95th Percentile	116.97	689.39	1,113.93	12.75	123.17
10th Decile Subcategory 10y	75th Percentile	102.17	172.15	195.92	3.36	17.91
Market Value of Equity Range	**50th Percentile**	**83.92**	**110.30**	**47.49**	**(5.93)**	**(1.30)**
$62.2 Million and $117.0 Million	25th Percentile	70.29	86.48	17.15	(25.52)	(20.22)
	5th Percentile	62.20	64.05	2.33	(48.27)	(38.52)
As of September 30, 2019:	95th Percentile	57.02	241.80	388.96	3.82	23.88
10th Decile Subcategory 10z	75th Percentile	41.79	60.47	66.76	(0.77)	2.43
Market Value of Equity Range	**50th Percentile**	**26.44**	**35.66**	**23.66**	**(6.85)**	**(3.06)**
$4.5 Million and $57.0 Million	25th Percentile	12.21	17.49	6.21	(17.11)	(11.35)
	5th Percentile	4.55	6.89	0.82	(29.79)	23.21

Sources: 2020 Cost of Capital Annual U.S. Guidence and Examples, Kroll Cost of Capital Navigator;*2017 Valuation Handbook: U.S. Guide to Cost of Capital* , Exhibit 4-10; and *2014 Valuation Handbook: Guide to Cost of Capital* , Exhibit 4-9.

Collectively, this information supports the theory that the CRSP size premium 10th decile is comprised of troubled and distressed companies.

According to James Hitchner in *Financial Valuation and Litigation Expert*, "It's important to note that 80 percent of the companies in decile category 10b are from 10z. As such, let's focus on 10z. At the 50th percentile of 10z the operating margin is –1.11 percent. Yes, on average, these companies are losing money. At the 25th percentile the operating margin is –21.27 percent. Furthermore, 62 percent of the companies in 10z are from only three industry sectors: financial services, technology, and healthcare."[35]

As indicated by Hitchner, based on dated information that is still relevant, not only does the CRSP size premium 10th decile include troubled companies, it is skewed by its industry concentration.

A few years back, Morningstar provided some additional detail related to the 10th decile regarding the probability of default of the companies in the 10th decile. Exhibit 6 provides statistics, as published in the *Ibbotson SBBI 2012 Valuation Yearbook* by Morningstar, of the probability of default of companies in the decile 10 subcategories.

As of December 31, 2011, a little less than 20 percent of subcategory 10b had a 25 percent probability of default. As company size decreases, from subcategory 10w to subcategory 10z, the probability of default increases.

As presented in the *Ibbotson SBBI 2013 Valuation Yearbook* published by Morningstar, the 10th decile

Exhibit 6
Probability of Default of the Decile 10 Subcategories
As of December 31, 2011

Probability of Default	10a Percent of Companies	10b Percent of Companies	10w Percent of Companies	10x Percent of Companies	10y Percent of Companies
75%	0	3	0	0	1
50%	2	7	1	3	3
25%	5	17	4	7	12
20%	6	21	4	7	14
15%	8	25	5	10	17
10%	10	31	8	13	22
5%	16	38	15	17	28

Source: *2012 Ibbotson SBBI Valuation Yearbook* , Table 7-15.

was comprised of significantly more companies in the 10b subcategory than the 10a subcategory.[36] As of December 31, 2002, there were 319 companies populating the 10a subcategory and 1,124 companies populating the 10b subcategory.

Furthermore, as of December 31, 2012, the significant majority of the 10b category was comprised of companies in the 10z subcategory—846 companies in 10z compared to 278 companies in 10y.[37]

Of these companies in the 10z subcategory, the majority were financial services businesses.[38]

Also, as presented in the *SBBI 2013 Valuation Yearbook*, Morningstar changed its methodology for determining the likelihood of company default.

The results of the new methodology were similar to the results of the methodology used for the *SBBI 2012 Valuation Yearbook*. Morningstar concluded that financial distressed companies are more likely to be small equity capitalization stocks.[39]

Liquidity May Be More Significant than Size in Assessing Risk

According to Aswath Damodaran, "the notion that market for publicly traded stocks is wide and deep has led to the argument that the net effect of illiquidity on aggregate equity risk premiums should be small."[40]

It is generally accepted that less liquid securities are inherently of a greater risk profile than highly liquid securities and, therefore, investors require greater rates of return to invest in less liquid investments. In fact, a growing body of work investigating the impact of liquidity on returns has emerged.[41]

The cost of illiquidity on security pricing is influenced by macroeconomic direction. Stock illiquidity increases when economies slow down and during periods of crisis, thus exaggerating the effects of both phenomena on the equity risk premium.[42]

Security liquidity has value as discussed in the following example. Consider two assets with the same cash flow and average liquidity, but one asset has much more liquidity risk . . . if the assets had the same price, investors would avoid the one with the high liquidity risk, because they would fear bearing greater losses if they needed to sell it in a liquidity crisis.[43]

For many analysts, the calculation of the cost of equity includes a size risk premium alpha factor developed from the CRSP database. There are numerous theories addressing why small market capitalization stocks provide greater investment returns.

However, there is an increasing amount of interest as to how the CRSP size risk premium decile conclusions may be skewed by an embedded liquidity discount.

Several studies have shown that an embedded stock liquidity discount helps to explain part of the reason that smaller capitalization companies generate higher returns—that is, the investor is compensated for investing in a low liquidity and, therefore, riskier asset.

Exhibit 7 presents liquidity statistics and the impact of liquidity organized by equity market

Exhibit 7
Liquidity Effect on the Size Risk Premium
Based on Quartile Portfolio Classifications for 2020
As Published in the Kroll Cost of Capital Navigator

	Low Liquidity	Mid-Low Liquidity	Mid-High Liquidity	High Liquidity	Liquidity Effect (%)
Micro-Cap					
Geometric Mean (%)	15.44	15.28	9.42	-0.65	16.09
Arithmetic Mean (%)	17.74	18.79	14.47	4.39	13.35
Standard Deviation (%)	22.54	28.36	34.05	32.81	
Average Number of Companies	348	181	122	96	
Small-Cap					
Geometric Mean (%)	15.25	14.22	11.91	5.69	9.56
Arithmetic Mean (%)	16.85	16.67	15.1	9.7	7.15
Standard Deviation (%)	19.19	23.43	26.57	29.72	
Average Number of Companies	198	201	173	175	
Mid-Cap					
Geometric Mean (%)	13.68	13.65	12.74	8.14	5.54
Arithmetic Mean (%)	15.01	15.31	14.8	11.56	3.45
Standard Deviation (%)	17.5	19.51	21.35	27.09	
Average Number of Companies	128	177	204	240	
Large-Cap					
Geometric Mean (%)	11.43	12.33	11.84	8.95	2.48
Arithmetic Mean (%)	12.64	13.45	13.35	11.81	0.83
Standard Deviation (%)	16.17	15.46	17.74	24.31	
Average Number of Companies	73	188	249	237	
Size Effect (%)	4.01	2.95	-2.42	-9.60	

Source: Cost of Capital: Annual U.S. Guidance and Examples, Kroll Cost of Capital Navigator, Exhibit 4.17.

capitalization quartile classification. The analysis corresponds to publicly traded securities in the 1972 to 2019 time frame.

An interesting aspect of the embedded liquidity issue is that market capitalization and illiquidity are not always correlated since there are small, liquid companies and large, illiquid ones in the market.[44]

However, based on the data presented in Exhibit 7, it appears that the smallest capitalization securities are affected by liquidity concerns far more than larger capitalization securities. It is also noteworthy that the subcategory of micro-cap stocks populated with the most companies, on average, was classified as low liquidity securities—a total of 348 companies.

In a research article published in the *Journal of Business Valuation and Economic Loss*, Frank Torchio and Sunita Surana studied the effect of liquidity on size premium calculations ("Torchio study").[45]

According to the Torchio study, a substantial portion of the size premium measurement reflects lack of liquidity. The Torchio study found that the lack of liquidity issue, an embedded liquidity issue, is problematic in certain fair value cases.

It is problematic because the application of the size premium—more specifically the application of the premium in small company valuations—may cause the fair value to be understated and may include an unintended valuation discount.

Exhibit 8
Liquidity Risk Premium Analysis
Based on the Torchio Study
Using CRSP Data from 1926 to 2010

SBBI Decile Group	Liquidity Level	Liquidity Risk Premium (return in excess of CAPM return) (%)	2011 Ibbotson SBBI Size Risk Premium (%)	Difference between Liquidity Premium and Size Premium (%)
1	High	-1.35	-0.38	**-0.97**
1	Low	0.13		0.51
2	High	-0.16	0.81	**-0.97**
2	Low	2.25		1.44
3	High	-0.05	1.01	**-1.06**
3	Low	2.88		1.87
4	High	0.07	1.20	**-1.13**
4	Low	3.25		2.05
5	High	0.57	1.81	**-1.24**
5	Low	4.01		2.20
6	High	-0.33	1.82	**-2.15**
6	Low	4.90		3.08
7	High	0.06	1.88	**-1.82**
7	Low	4.34		2.46
8	High	0.19	2.65	**-2.46**
8	Low	5.40		2.75
9	High	1.99	2.94	**-0.95**
9	Low	5.25		2.31
10	High	2.46	6.36	**-3.90**
10	Low	11.18		4.82
10w	High	-0.37	3.99	**-4.36**
10w	Low	8.08		4.09
10x	High	4.57	4.96	**-0.39**
10x	Low	10.40		5.44
10y	High	3.34	9.15	**-5.81**
10y	Low	12.85		3.70
10z	High	3.57	12.06	**-8.49**
10z	Low	17.55		5.49

Source: Frank Torchio and Sunita Surana, "Effect of Liquidity on Size Premium and its Implications for Financial Valuations," *Journal of Business Valuation and Economic Loss* 9, no. 1 (2014): Tables 10, 11, and 12.

In order to study the effect of embedded liquidity related to the size risk premium, the Torchio study progressed through several procedures.[46] The three primary procedures are described as follows.

For the first procedure, the Torchio study replicated the Ibbotson SBBI 10 decile analysis using the CRSP database. The study applied the same or similar procedures used by Ibbotson, Duff & Phelps, and now Kroll to replicate the published SBBI 10 decile study results. It also replicated the 10th decile subcategories.

For the second procedure, the Torchio study subdivided the SBBI 10 deciles and 10th decile subcategories into high liquidity and low liquidity categories.

For the final procedure, the liquidity premium is calculated much the same way that the SBBI 10 decile size risk premiums are calculated. The liquidity premium is calculated as the excess return to the predicted CAPM return.

Exhibit 8 presents the Torchio study liquidity risk premium analysis results.[47]

The Torchio study provides empirical evidence of the impact that liquidity has on security rates of return. Based on Exhibit 8, the following conclusions appear to be true:

- The high liquidity level securities (stocks that exhibit trading liquidity above the decile group median) rates of return are significantly lower than the low liquidity level securities at each decile grouping.
- Compared to the size premium statistics presented in the *SBBI 2011 Valuation Yearbook*, the high liquidity group for each decile and subdecile category had much lower rates of return.
- For SBBI deciles 1 through 9, the difference between the high liquidity equity risk premium estimate and the SBBI size risk premium is not as significant as it is for decile 10 and subcategories.

- The liquidity risk premium effect is most pronounced at the 10z subcategory decile.
- The size risk premium is clearly influenced by the low liquidity securities.

According to the Tochio study, the large-size premiums calculated by Ibbotson are the consequence of a disproportionately greater number of low liquidity stocks comprising the small-size portfolios.[48]

For fair value measurements in certain jurisdictions due to the presence of an embedded liquidity discount, the application of an equity size risk premium alpha factor based on the 10th decile or 10th decile subcategories may not be appropriate.

Summary and Conclusion

Damages analysts routinely develop damages measurements that include income projections and apply a present value discount rate. Such damages measurements include lost profits method analyses. Such damages measurements also include cost to cure method analyses and reasonable royalty rate method analyses.

One of the typical components of the damages analysis discount rate is the estimation of cost of equity capital. The measurement of many of the cost of equity capital components are typically not controversial in the damages measurement discount rate calculation. The measurement of the size risk premium component of the cost of equity capital can sometimes result in disagreement with regard to the discount rate calculation.

This discussion summarizes many of the damages analyst considerations with regard to the measurement of the equity size risk premium. This equity size risk premium is one component of the present value discount rate calculation developed as part of the damages measurement analysis.

The focus of this discussion was to provide some background and information on the components related to the measurement of the cost of equity capital. In particular, this discussion focused on the measurement of the size risk premium component.

Dating back to the Banz study, and more recently by way of the Kroll CRSP size risk premium analysis, empirical evidence has been gathered and analyzed in support of the size-related phenomena theory. Small private company investment returns cannot be entirely explained by the standard application of the basic CAPM model for estimating the cost of equity capital.

Because the basic CAPM does not entirely explain small private company investment returns, analysts typically apply the MCAPM to estimate the cost of equity capital in such instances.

There are many observations regarding the size-related phenomena theory and the CRSP size risk premium data used by damages analysts. These observations include the following:

1. The small capitalization premium has disappeared in recent years. The empirical evidence supports varying size-related premium at different points in time. Therefore, in certain time periods, it would not be surprising for small capitalization stocks to provide lower investment returns than larger capitalization stocks.
2. The premium, at the smallest level, is unduly influenced by stocks of less than $5 million in market capitalization and stocks that trade at prices less than $2 per share. The most statistical noise in the CRSP size premium data is in the 10th decile classification and its smaller subcategory classifications. This factor may not be as relevant if the subject matter company is a very small business that is similar to the companies that populate the 10th subcategories of 10y and 10z.
3. Other factors, specifically liquidity or lack thereof, provide important detail that analysts should consider in the decision to use, or not to use, the CRSP size risk premium data.

The application of a size risk premium in the development of the cost of equity capital is considered a generally accepted procedure for many damages measurement methods. However, damages analysts should be aware of the above-described issues related to the application of a size risk premium to develop a discount rate for a damages measurement analysis.

Notes:

1. There are many other cost of equity capital estimation models including (a) the Kroll, Risk Premium Report Model; (b) arbitrage pricing theory model; and (c) Fama-French three factor model.
2. CRSP is an acronym for Center for Research in Security Prices. The *Valuation Handbook* is a continuation of the previously produced *SBBI Valuation Yearbook* by Morningstar. The *Valuation Handbook* is produced by Duff & Phelps (a Kroll business).
3. Unsystematic risk is defined as the portion of total risk that is specific to an individual security and can be avoided through diversification.

Shannon P. Pratt, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 5th ed. (New York: McGraw-Hill, 2008), 1075, Appendix A.

4. CAPM is defined as a model in which the cost of capital for any stock or portfolio of stocks equals a risk-free rate plus a risk premium that is proportionate to the systematic risk of the stock or portfolio. Pratt, *Valuing a Business*, 1070, Appendix A.
5. Beta is defined as a measure of the systematic risk of a stock; the tendency of a stock's price to be correlated with changes in a specific index. Pratt, *Valuing a Business*, 1070, Appendix A.
6. Roger J. Grabowski, "The Size Effect—It Is Still Relevant," *Business Valuation Review* 35, no. 2 (Summer 2016): 63.
7. Rolf W. Banz, "The Relationship between Return and Market Value of Common Stocks," *Journal of Financial Economics* 9 (March 1981): 3–18.
8. The Cost of Capital Navigator is found at: www.kroll.com/en/cost-of-capital.
9. The Cost of Capital Navigator presents an alternative size premium analysis, the Risk Premium Report. The Risk Premium Report is not discussed herein.
10. Annual stock market returns represent the combined annual stock returns of stocks listed on the New York Stock Exchange ("NYSE"), NYSE Euronext, and Nasdaq.
11. The standard deviation is a measure that is used to quantify the amount of variation or dispersion of a set of data values. A low standard deviation indicates that the data points tend to be close to the mean of the set, while a high standard deviation indicates that the data points are spread out over a wider range of values. J.M. Bland and D.G. Altman, "Statistics Notes: Measurement Error," *The BMJ* 312 (7047) (September 1996): 1654.
12. Aswath Damodaran, "Equity Risk Premiums (ERP): Determinants, Estimation and Implications—The 2015 Edition," Stern School of Business whitepaper (March 2015): 37.
13. Ibid.
14. Aswath Damodaran, "The Small Cap Premium: Where Is the Beef?" *Business Valuation Review* 34, no. 4 (Winter 2015): 153.
15. Joel L. Horowitz, Tim Loughran, and N.E. Savin, "The Disappearing Size Effect," *Research in Economics* 54, no. 1 (2000): 87.
16. Ibid.
17. Ibid.: 96.
18. Duff & Phelps, *2017 Valuation Handbook: U.S. Guide to Cost of Capital*, 4–6.
19. Grabowski, "The Size Effect—It Is Still Relevant": 65.
20. Ibid.
21. Damodaran, "The Small Cap Premium: Where Is the Beef?": 154.
22. Horowitz, Loughran, and Savin, "The Disappearing Size Effect."
23. Damodaran, "The Small Cap Premium: Where Is the Beef?": 154.
24. Ibid.
25. Roger Grabowski and Shannon Pratt, *Cost of Capital*, 5th ed. (New York: John Wiley & Sons, 2014), 363.
26. Horowitz, Loughran, and Savin, "The Disappearing Size Effect": 87.
27. Grabowski and Pratt, *Cost of Capital*, 364.
28. Ibid., 365.
29. Ibid.
30. Ibid., 366.
31. The Kroll Cost of Capital Navigator.
32. Ibid.
33. Ibid.
34. Ibid.
35. Jim Hitchner, "How to 'Rig' a Valuation: The Discount Rate," *Financial Valuation and Litigation Expert* (February/March 2013).
36. *2013 Ibbotson SBBI Valuation Yearbook* (Chicago: Morningstar, 2013), 90.
37. Ibid.
38. Ibid., 91.
39. Ibid., 100.
40. Damodaran, "Equity Risk Premiums (ERP): Determinants, Estimation and Implications—The 2015 Edition": 12.
41. Duff & Phelps, *2017 Valuation Handbook: U.S. Guide to Cost of Capital*, 4–21.
42. Damodaran, "Equity Risk Premiums (ERP): Determinants, Estimation and Implications—The 2015 Edition": 12.
43. Yakov Amihud, Haim Mendelson, and Lasse Heje Pedersen, *Market Liquidity, Asset Pricing, Risk, and Crises* (Cambridge: Cambridge University Press, 2013), 103.
44. Damodaran, "The Small Cap Premium: Where Is the Beef?"
45. Frank Torchio and Sunita Surana, "Effect of Liquidity on Size Premium and Its Implications for Financial Valuations," *Journal of Business Valuation and Economic Loss* 9, no. 1 (2014): 55–85.
46. The Tochio study was based on monthly stock data provided by the CRSP database for the period of 1926 to 2010.
47. Torchio and Surana, "Effect of Liquidity on Size Premium and its Implications for Financial Valuations": 77–79.
48. Ibid.: 77.

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